# Core-Selecting Auctions for Autonomous Vehicle Public Transportation System

James J.Q. Yu, *Member, IEEE,* and Albert Y.S. Lam, *Senior Member, IEEE*

***Abstract*—With the technological advancements, the autonomous vehicle (AV) is expected to play an active role in the future transportation system. An AV-based public transportation system was recently proposed to unleash the full capability of AVs to provide effective and flexible transportation services. It establishes a new public transportation market which can accommodate multiple vehicle operators. Such multi-tenant system encourages market competition for better quality of service. The pricing process in the original proposal was designed based on the Vickrey-Clarke-Groves mechanism, but it is vulnerable to the problems of payoff reduction and shill-bidding. In this paper, we re-investigate the pricing process to address these vulnerabilities. We formulate it as core-selecting reverse combinatorial auctions and investigate the properties of the "core". We establish a core-selecting mechanism which can maximize customer's utility and prevent shill-bidding. We analyze the theoretical properties of the formulated auctions and the core-selecting mechanism. We verify the results with extensive simulations. The simulation results show that the core-selecting mechanism can result in lower service charge, and suppress untruthfulness, shill-bidding, and coalition formation. It can produce auction results in linear computation time, making it scalable and practical.**

***Index Terms*—Autonomous vehicle, reverse combinatorial auction, public transportation system, core-selecting auction.**

## I. INTRODUCTION

The smart city is believed to be an important urban development vision toward improving quality of life based on the Internet of Things backbone [1]. It can facilitate intelligent resource utilization and it is a promising solution to many problems due to large-scale urbanization, e.g., serious traffic congestion and air pollution [2]–[5]. Among those empowering technologies, intelligent transportation system (ITS) is one of the essential components for accommodating a massive volume of transportation demand with limited damage to the environment. One may envision that ITS can help enhance the current transportation system with high capacity and safety through improving the transport carriers and backend management system. Driven by the technological advancements and consumer preference, carriers undergo a gradual transition from internal combustion engine to electric vehicles, and from manned to autonomous cars [6]. However, the supporting management system demands more research effort to achieve the visions of smart city.

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J.J.Q. Yu is with the HKU Shenzhen Institute of Research and Innovation (SIRI), Shenzhen 518057, China, and the Department of Electrical and Electronic Engineering, The University of Hong Kong, Hong Kong (e-mail: jqyu@eee.hku.hk).

A.Y.S. Lam is with Fano Labs and the Department of Electrical and Electronic Engineering, The University of Hong Kong, Hong Kong (e-mail: albert@fano.ai).

The autonomous vehicle (AV), known for its driverless and environmentally friendly characteristics, is gaining much attention of the public and the research community. On the one hand, an AV can adapt to road conditions and determine an optimal driving pattern [7], [8]. This feature brings significant advantages, including fewer traffic collisions and effective transportation. On the other hand, AVs are usually connected and can communicate with each other to acquire various system information [9]. A central control center can be employed to collectively decide the routes and schedules of AVs to improve social welfare. This grants AVs higher controllability than the traditional vehicles [9]. This full-fledged controllability improves traffic flow [10], without heavily relying on the traffic assistant facilities (e.g., road signage and traffic light [11]). Reduced traffic congestion and faster pilot speed can also result in better fuel efficiency and lower carbon footprint.

Recently, a new AV-based public transportation system, called Autonomous Vehicle Public Transportation System (AVPTS) has been developed, where AVs serve as transportation carriers [6], [12]. In this system, a control center is established to manage a fleet of AVs to provide on-demand point-to-point transportation services. Customers can place transportation requests with specific pickup locations and destinations via e-hailing. The control center checks the admissibility of requests and allocates appropriate AVs to serve the passengers. Through coordinated routing and scheduling, we can maximize operational profit and transportation capacity. Contributed by AVs, AVPTS is expected to provide a precise, safe, effective, intelligent, and economically favorable way to cater point-to-point transportation services with optional ride sharing [12].

While the control center is dedicated to the coordination and administrative tasks in AVPTS, it is possible to have more than one service operator managing the vehicles. In fact, adequate market competition may lead to better service charge and quality, and thus it is beneficial to the future smart city to recruit multiple AV operators to form an AV public transportation market. Such a market requires a proper pricing mechanism for the services, which should be fair for all governed operators and at the same time maximize the utility of the customers. [13] proposed a Vickrey-Clarke-Groves (VCG) [14]–[16] auction-based pricing scheme for AVPTS. In this design, the control center is responsible for hosting the auction. Upon receiving a transportation request, the AV operators evaluate their own operational costs for the service and propose the corresponding service charges to the control center. After gathering all the bids, the control center determines the winners of the auction and then set their actual service charges using the VCG mechanism, which is further discussed in Appendix A.

VCG is widely known as the only auction mechanism to be both truthful and efficient [17], but it suffers from some problems including payoff reduction and shill-bidding. This mechanism tends to generating lower payoff for the auctioneer, which, in our case, stands for a higher customer service charge. It is also susceptible to a form of strategic bidding, known as "shill-bidding" or "false-name bidding", where a bidder impersonates multiple bidders to unilaterally increase its own utility and undermine the social welfare [18]. An alternative implementation of shill-bidding is that several bidders can collude to form a coalition for profit. This vulnerability is further illustrated in the example given in Appendix A.

As an alternative, the core-selecting auction is considered as a promising pricing mechanism to overcome those vulnerabilities brought by VCG while maintaining the bidders' truthfulness [19], [20]. With proper design, the core-selecting-based pricing mechanism can eliminate the participants' motivations of seceding and forming a coalition for their own profit. Core-selecting auctions have been applied to several engineering disciplines. For example, [21] employs the core-selecting auctions in a cognitive radio system to allocate radio spectrums to users, and [22] uses the mechanism in cloud computing to sell computing power to customers. There are also real-world implementations. For example, core-selecting auctions were employed in primary spectrum markets of the United Kingdom and other countries [23]. However, all these only focus on the ordinary auction where the bidders buy items from the auctioneer. However, the service charge determination of this work fits into a reverse auction where the bidders sell items to the auctioneer. So the mechanism needs to be re-investigated for AVPTS.

In this work, we formulate several reverse combinatorial auctions for the multi-tenant AVPTS, in which each AV in the system is considered as a bidder. We employ the core-selecting auction mechanism and prove its robustness against shills and coalitions in the reverse combinatorial auctions. We further design a quadratic core-selecting charging rule to maximize the bidders' incentive for truth-telling and the benefit of customers. The contributions of this work include the following:

- We design a core-selecting auction for the multi-tenant AVPTS [6], [12], and study its theoretical properties.
- We propose an implementation of the quadratic pricing rule to maximize the operators' incentive to report bids truthfully.
- We conduct comprehensive case studies to verify our analytical results and evaluate the efficacy of core-selecting auctions.

This paper is outlined as follows. We introduce the system model of AVPTS and its multi-tenant variant, and provide the formulations of AV public transportation reverse combinatorial auctions in Section II. In Section III, we present our core-selecting auction design and investigate its theoretical properties. Section IV presents the quadratic core-selecting charging rule and discusses its practical implementation. Extensive simulations are demonstrated in Section V, and Section VI concludes the paper.

## II. System Model

In this section, we first introduce the model of multi-tenant AVPTS and elaborate its pricing process. Then we explain the reverse combinatorial auctions for the multi-tenant AVPTS.

### A. Multi-tenant AVPTS

The original design of AVPTS assumes that all vehicles are governed by the same operator, i.e., the system *per se*. The primitive system has a control center which coordinates a fleet of AVs to provide transportation services. Customers submit transportation requests to the control center with necessary transportation information, e.g., pickup and destination locations, number of passengers, service time requirements, etc [24]. Upon receipt of a transportation request, the control center verifies its admissibility through the admission control process [12], [25]. Once a request is admitted, appropriate AVs are assigned to implement the service. There are three types of transportation services in AVPTS, as defined in [13]: (i) splittable, (ii) non-splittable, and (iii) private services. For (i), passengers belonging to a single request may be split into groups, each served by a separate vehicle. It also allows passengers from different requests sharing the same ride. For (ii), each request must be served by one vehicle, where ridesharing is allowed. For (iii), passengers of a request should occupy the whole vehicle during the ride. To implement ridesharing, we need to jointly determine the routes and schedules of several AVs to accommodate multiple requests. This can be achieved through the scheduling process [12], which can minimize the overall system cost.

This system lays the foundation of adopting AVs in a public transportation system. Meanwhile, one system operator is assumed to determine the charge of service resulting in the formation of monopoly. This encourages the only service provider to manipulate the service charges, and poor service quality and lack of customer sovereignty can be expected. In practice, many modern deregulated public transportation systems, such as taxis, embrace multiple business entities (i.e., operators) to enhance the social welfare, see [26] for an example. Therefore, a multi-tenant AVPTS is developed in which multiple AV operators are allowed to compete to provide service to the customers [13]. In this system, a tenant refers to an operator. Vehicles governed by an operator are considered cooperative while there are likely competitions among the operators for economic reasons. When more than one operator are interested in offering service to a request, a pricing process is involved to settle the service charge. We illustrate the pricing process in Fig. 1. In practice, customers submit service requests to a broker (Step 1), who is the middleman responsible for arbitrating the admission competition among the operators. Once receiving a request, the broker disseminates its details to the operators (Step 2), each of which then assesses its operational cost through the scheduling process (Step 3) [12]. Based on this cost, each operator will publish its proposed service charge to the broker (Step 4), who will determine the winner and return the settled charge to the customer for decision (Step 5).

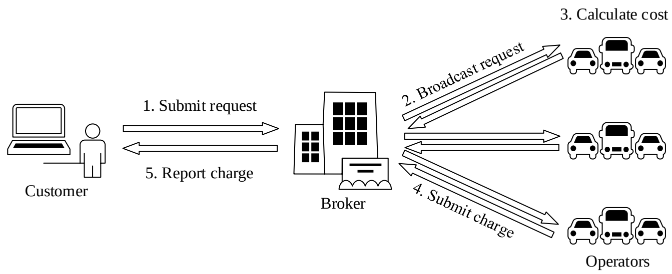



Fig. 1. Multi-operator AV public transportation system. Numbers indicate the pricing process.

### B. Reverse Combinatorial Auction

We basically follow [13] to model the pricing process as an auction. In this auction, seat occupancy is regarded as an item to be sold. Each AV acts as a bidder, and those AVs belonging to the same operator may place bids cooperatively. An auctioneer (broker) determines the winning AVs based on the submitted bids. Auctioning multiple items for sell at the same time constitutes a reverse combinatorial auction [27], [28]. Therefore, the pricing process is actually a reverse combinatorial auction, which is called the AV public transportation auction in the sequel.

Consider a transportation request with $r \in \mathbb{N}$ passengers. Let $\mathcal{K}$ be the set of AVs. AV $k \in \mathcal{K}$ has $\overline{q}_k$ seats and there are $q_k \leq \overline{q}_k$ seats available.[1] Without loss of generality, we assume [2]

$$r \leq \overline{q}_k, \forall k \in \mathcal{K}, \tag{1}$$

and all seats are homogeneous. Hence $k$ has a collection of seat combinations $\mathcal{Q}_k = \{\{1\}, \{1,2\}, \dots, \{1,\dots,q_k\}\}$ for lease. Based on the operational cost computed through the scheduling process, each participating operator has its own valuation on each seat combination $\mathcal{S} \in \mathcal{Q}_k$ for its governing AV $k$, denoted by $v_k(\mathcal{S})$. The size of $\mathcal{S}$ is given by $s$, i.e., $s = |\mathcal{S}|$. Consequently, each bidder can submit at most $q_k$ different bids $b_k(\mathcal{S})$ for leasing $s$ seats in AV $k$. We assume that all operators are rational, and thus the value of a bid is no less than its true valuation, i.e., $b_k(\mathcal{S}) \geq v_k(\mathcal{S}) \geq 0$. We define a quasi-linear utility $u_k$ for $k$ as

$$u_k = \begin{cases} c_k - v_k(\mathcal{S}_k) & k \text{ wins the auction,} \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where $\mathcal{S}_k$ is the set of seat occupancies that $k$ has committed after the auction. $c_k$ is the actual service charge by $k$, which is determined through a charging rule discussed in Section IV.

We define the binary variable $x_k(\mathcal{S}) \in \{0,1\}$ to indicate whether $k$ wins the auction with $\mathcal{S}$. For the benefit of the customer, we aim to determine a combination of seat occupancies with the lowest total charge to accommodate $r$ passengers and thus we minimize the total sum of bids. The Winner Determination Problem (WDP) for splittable service can be formulated as follows [13]:

$$\text{minimize} \sum_{k\in\mathcal{K}} \sum_{\mathcal{S}\in\mathcal{Q}_k} x_k(\mathcal{S}) b_k(\mathcal{S}) \tag{3a}$$

$$\text{subject to} \sum_{\mathcal{S}\in\mathcal{Q}_k} x_k(\mathcal{S}) \leq 1, \forall k \in \mathcal{K} \tag{3b}$$

$$\sum_{k\in\mathcal{K}} \sum_{\mathcal{S}\in\mathcal{Q}_k} s x_k(\mathcal{S}) \geq r. \tag{3c}$$

(3b) ensures that each AV can at most win for one bid. This guarantees that no seat can be occupied more than once. It can be relaxed if the bidding function is concave and we will elaborate this in Lemma 1 later. (3c) guarantees that enough seats can be reserved for the customer.

For the non-splittable service, only one vehicle is allowed to serve the request and thus there is only one winner. Based on (3), the corresponding WDP is defined as follows [13]:

$$\text{minimize} \sum_{k\in\mathcal{K}} \sum_{\mathcal{S}\in\mathcal{Q}_k} x_k(\mathcal{S}) b_k(\mathcal{S}) \tag{4a}$$

$$\text{subject to} \sum_{k\in\mathcal{K}} \sum_{\mathcal{S}\in\mathcal{Q}_k} x_k(\mathcal{S}) = 1, \tag{4b}$$

$$\sum_{k\in\mathcal{K}} \sum_{\mathcal{S}\in\mathcal{Q}_k} s x_k(\mathcal{S}) \geq r. \tag{4c}$$

The only difference between (3) and (4) lies in their first constraints. (4b) ensures that there is only one winner.

For the private service, an extra constraint is required to ensure that the request is served by an empty vehicle. Therefore the respective WDP is defined as [13]:

$$\text{minimize} \sum_{k\in\mathcal{K}} \sum_{\mathcal{S}\in\mathcal{Q}_k} x_k(\mathcal{S}) b_k(\mathcal{S}) \tag{5a}$$

$$\text{subject to} \sum_{k\in\mathcal{K}} \sum_{\mathcal{S}\in\mathcal{Q}_k} x_k(\mathcal{S}) = 1, \tag{5b}$$

$$\sum_{k\in\mathcal{K}} \sum_{\mathcal{S}\in\mathcal{Q}_k} s x_k(\mathcal{S}) \geq r. \tag{5c}$$

$$s x_k(\mathcal{S}) = \overline{q}_k x_k(\mathcal{S}), \forall k \in \mathcal{K}, \forall Q \in \mathcal{Q}_k. \tag{5d}$$

If an AV $k$ wins with $\mathcal{S}$, (5d) guarantees that $s = \overline{q}_k$, which means that the number of seats provided is equal to the vehicle capacity.

**Lemma 1.** *Each AV can at most have one winning bid for each service type if the bidding function $b_k(\cdot)$ is concave.*

*Proof.* It is trivial for the non-splittable and private services. For the splittable service, suppose that AV $k$ wins the auction with two bids $b_k(\mathcal{S}')$ and $b_k(\mathcal{S}^*)$, for $\mathcal{S}', \mathcal{S}^* \in \mathcal{Q}_k$. As the auction tries to minimize total service charge, the customer can instead select another bid $b_k(\mathcal{S})$ such that the sizes of these bids satisfy $|\mathcal{S}| = |\mathcal{S}'| + |\mathcal{S}^*|$, and $b_k(\mathcal{S}) < b_k(\mathcal{S}') + b_k(\mathcal{S}^*)$. This induces a contradiction. ■

**Lemma 2.** *Constraints* (3c) *and* (4c) *become* $\sum_{k\in\mathcal{K}} \sum_{\mathcal{S}\in\mathcal{Q}_k} s x_k(\mathcal{S}) = r$ *if* $b_k(\cdot)$ *is increasing.*

[1] $\overline{q}_k - q_k$ seats in $k$ have been reserved for another request.

[2] If $r > \overline{q}_k$, the request can be split into multiple ones so that (1) is satisfied for each split request.0

*Proof.* This lemma can be proved by contradiction. Consider that $\tilde{k}$ wins the auction with bid $b_{\tilde{k}}(\mathcal{S}_{\tilde{k}})$, $\mathcal{S}_{\tilde{k}} \in \mathcal{Q}_{\tilde{k}}$. Suppose $\sum_{k\in\mathcal{K}}\sum_{\mathcal{S}\in\mathcal{Q}_k} sx_k(\mathcal{S}) > r$, then we have

$$\begin{aligned}&|\mathcal{S}_{\tilde{k}}| + \sum_{k\in\mathcal{W}\setminus\tilde{k}} |\mathcal{S}_k| = |\mathcal{S}_{\tilde{k}}| + \sum_{k\in\mathcal{W}\setminus\tilde{k}}\sum_{\mathcal{S}\in\mathcal{Q}_k} sx_k(\mathcal{S})\\ =&\sum_{\mathcal{S}\in\mathcal{Q}_{\tilde{k}}} sx_{\tilde{k}}(\mathcal{S}) + \sum_{k\in\mathcal{K}\setminus\tilde{k}}\sum_{\mathcal{S}\in\mathcal{Q}_k} sx_k(\mathcal{S}) = \sum_{k\in\mathcal{K}}\sum_{\mathcal{S}\in\mathcal{Q}_k} sx_k(\mathcal{S}) > r,\end{aligned}$$

where $\mathcal{W} \subseteq \mathcal{K}$ is the set of winners. Here we consider two cases:

1) $|\mathcal{S}_{\tilde{k}}| > 1$. In this case the auctioneer can select another bid submitted by $\tilde{k}$, denoted by $b_{\tilde{k}}(\mathcal{S}'_{\tilde{k}})$, such that $|\mathcal{S}'_{\tilde{k}}| = |\mathcal{S}_{\tilde{k}}| - 1$. As the provided seat occupancies are greater than the requested, this bid always exists. Therefore we have (all terms in integer):

$$|\mathcal{S}_{\tilde{k}}| + \sum_{k\in\mathcal{W}\setminus\tilde{k}} |\mathcal{S}_k| > |\mathcal{S}'_{\tilde{k}}| + \sum_{k\in\mathcal{W}\setminus\tilde{k}} |\mathcal{S}_k| \geq r. \tag{6}$$

The auctioneer should favor $\mathcal{S}'_{\tilde{k}}$ over $\mathcal{S}_{\tilde{k}}$ as the former requires a smaller charge. This contradicts with the assumption that $k$ wins with $\mathcal{S}_{\tilde{k}}$.

2) $|\mathcal{S}_{\tilde{k}}| = 1$. In this case the auctioneer should prefer another set of winners $\mathcal{W} \setminus \tilde{k}$ where other winners commit the same seat occupancies. By removing $\tilde{k}$ from the winners, we still get $\sum_{k\in\mathcal{K}}\sum_{\mathcal{S}\in\mathcal{Q}_k} sx_k(\mathcal{S}) \geq r$, and the total service charge is reduced. This contradicts with the assumption that $\tilde{k}$ is a winner. ■

Lemma 2 tightens an inequality constraint in (3) and (4) and this can significantly reduce the solution spaces of the optimization problems, and may potentially lead to faster computation speed. However, Lemma 2 cannot be applied to the private service (5) as each AV can only have one feasible bid due to 5d. Therefore the auctioneer cannot find another bid to satisfy the constraints while incur a smaller charge.

## III. Core-Selecting AV Public Transportation Auctions

In this section, we study the properties of the core of AV public transportation auctions, aiming to develop robust outcomes against shill bidding and capable of achieving near-optimal utility to the customer. We first define a general form of the core for our reverse combinatorial auctions, followed by the advantages of introducing core-selecting mechanism to the AV public transportation auctions. Note that we study the integration of core-selecting mechanism with AVPTS in this paper. As the proposed auctioning mechanism is not limited to AVPTS, It can be applied to other transportation systems with proper modifications, which is beyond the scope of this work.

### A. The core

The core represents a collection of satisfactory outcomes from an auction. It has some unique properties which allow us to prevent shill bidding. Let $\pi_k = c_k - b_k(\mathcal{S}_k)$ be the observable surplus of $k$ when $k$ is a winner of the auction and commits to provide $\mathcal{S}_k$ for the service, and $\pi_k = 0, \forall k \notin \mathcal{W}$ where $\mathcal{W} \subseteq \mathcal{K}$ is the set of winners. Instead of the true utility $u_k$ given in (2), the core is defined over $\pi_k$ because the auctioneer does not have any knowledge on the bidders' real valuation $v_k(\mathcal{S})$ without a guarantee on the incentive compatibility [29]. We use "0" to denote the auctioneer, and define the customer's (auctioneer's) utility as $u_0 = \pi_0 = v_0 - \sum_{k\in\mathcal{K}} c_k$, where $v_0$ is the customer's valuation of the service. $v_0$ can be set to a sufficient large value to ensure $u_0 \geq 0$ without affecting the outcomes of the auction. $v_0$ is considered to be known to the auctioneer but not the bidders for fairness concerns.

**Definition 1** (Core)**.** *An imputation $\boldsymbol{\pi} = \{\pi_k | k \in \mathcal{K}\cup\{0\}\}$ is a feasible non-negative observable surplus profile. An imputation $\boldsymbol{\pi}$ is blocked by coalition $\mathcal{C} \subseteq \mathcal{K}$ if there is another $\boldsymbol{\pi}'$ such that $\pi'_k \geq \pi_k$ for all $k \in \mathcal{C}$, and $\pi'_0 > \pi_0$. The core is the set of imputations that are feasible and not blocked by any coalition.*

This implies that when the auctioneer selects a profile from the core, there is no incentive for any bidder $k$ to form a coalition $\mathcal{C}$ with other bidders for any possible improvement of their total utility. This properties renders it meaningless for the bidders to bid with false names.

For simplicity and by abuse of notation, we use $\alpha(\mathcal{C})$ to represent the optimal objective value of WDP (3), (4), or (5), depending on the service type, where only bidders in $\mathcal{C}$ participate in the auction. We further define $w(\mathcal{C}) \triangleq v_0 - \alpha(\mathcal{C})$, whose practical meaning is given as follows:

**Proposition 1.** *$w(\mathcal{C})$ is the total observable surplus of $\mathcal{C}\cup\{0\}$ in an auction when $\mathcal{C}$ are the only bidders.*

*Proof.* We use $\pi_k^{\mathcal{C}}$ to denote the observable surplus of $k \in \mathcal{C}\cup\{0\}$. Let $\mathcal{W}^{\mathcal{C}} \subseteq \mathcal{C}$ be the set of winners and $c_k^{\mathcal{C}}$ be the service charge of $k$. We have

$$\begin{aligned} w(\mathcal{C}) &= v_0 - \alpha(\mathcal{C}) = v_0 - \min\sum_{k\in\mathcal{C}}\sum_{\mathcal{S}\in\mathcal{Q}_k} x_k(\mathcal{S}) b_k(\mathcal{S})\\ &= v_0 - \sum_{k\in\mathcal{W}^{\mathcal{C}}} b_k(\mathcal{S}_k)\\ &= v_0 - \sum_{k\in\mathcal{W}^{\mathcal{C}}} c_k^{\mathcal{C}} + \sum_{k\in\mathcal{W}^{\mathcal{C}}} (c_k^{\mathcal{C}} - b_k(\mathcal{S}_k))\\ &= \pi_0^{\mathcal{C}} + \sum_{k\in\mathcal{W}^{\mathcal{C}}} \pi_k^{\mathcal{C}} = \pi_0^{\mathcal{C}} + \sum_{k\in\mathcal{C}} \pi_k^{\mathcal{C}}. \end{aligned}$$

■

Hence the core can be defined mathematically as

$$\begin{aligned}\mathrm{Core}(\mathcal{K}) = \{\boldsymbol{\pi} \geq 0 | &\sum_{k\in\mathcal{K}\cup\{0\}} \pi_k = w(\mathcal{K}),\\ &\sum_{k\in\mathcal{C}\cup\{0\}} \pi_k \geq w(\mathcal{C}), \forall \mathcal{C} \subseteq \mathcal{K}\},\end{aligned} \tag{7}$$

which follows Definition 1. With the first constraint in (7), we focus on those profiles with the optimal total utility. The second constraint emphasizes that the final payoff must no less than that induced from any possible coalition. As $w(\mathcal{C})$ is the maximum total surplus that coalition $\mathcal{C}\cup\{0\}$ can achieve without other bidders, a greater total surplus value (left-hand-side) prevents the existence of any blocking coalition.

In a first price auction, the winning bidders always charge the customer the values of their bids.

**Lemma 3.** *The charge profile of first price auction is in the core of AV public transportation auction.*

*Proof.* Let $\mathcal{W} \subseteq \mathcal{K}$ be the winners. For $k \in \mathcal{W}$, we will get $c_k = b_k(\mathcal{S}_k)$ if $k$ wins in the first price auction. Therefore, we have $\pi_k = 0$ for all $k \in \mathcal{K}$. We also have

$$\begin{aligned}\sum_{k\in\mathcal{K}\cup\{0\}} \pi_k = \pi_0 &= v_0 - \sum_{k\in\mathcal{W}} c_k \\ &= v_0 - \sum_{k\in\mathcal{W}} \sum_{\mathcal{S}\in\mathcal{Q}_k} x_k(\mathcal{S}) b_k(\mathcal{S}) \\ &= \max[v_0 - \sum_{k\in\mathcal{K}} \sum_{\mathcal{S}\in\mathcal{Q}_k} x_k(\mathcal{S}) b_k(\mathcal{S})] = w(\mathcal{K}),\end{aligned} \tag{8}$$

As $\alpha(\cdot)$ is the optimal objective value of a minimization problem, reducing $\mathcal{K}$ to $\mathcal{C}$ will make $\alpha(\mathcal{K}) \leq \alpha(\mathcal{C})$. Thus, we have

$$\begin{aligned}\sum_{k\in\mathcal{C}\cup\{0\}} \pi_k = \pi_0 = w(\mathcal{K}) &= v_0 - \alpha(\mathcal{K}) \\ &\geq v_0 - \alpha(\mathcal{C}) = w(\mathcal{C}), \forall \mathcal{C} \subseteq \mathcal{K}.\end{aligned} \tag{9}$$

Therefore, the first price auction service charge profile is in the core. ■

**Corollary 1.** *The core of AV public transportation auction is non-empty.*

We can see that an efficient charging mechanism should always produce an in-core auction result.

### *B. Incentive to Truth-telling*

The profiles in the core guarantee that no auction participants will form coalitions for better unilateral utilities. This makes the core-selecting auction robust against shill bidding. The core-selecting auction for AVPTS is defined as follows:

**Definition 2** (Core-selecting AV public transportation Auction)**.** *A core-selecting AV public transportation auction is an auction that always results in* $\boldsymbol{\pi} \in \mathrm{Core}(\mathcal{K})$ *such that the customer pays the minimum possible service charge, i.e., maximum customer utility, within the core:*

$$\pi_0^* = \max\{\pi_0 | \boldsymbol{\pi} \in \mathrm{Core}(\mathcal{K})\}. \tag{10}$$

By definition, the core consists of all dominating profiles of the bidders with observable surplus and they cannot be further improved without undermining any participant's utility. However, it is still doubtful whether the core-selecting auction can develop satisfactory customer utility, i.e., minimal service charge, in the presence of shill bids. As VCG is the only truthful and efficient auction mechanism [17], [30], the closer the profile with observable surplus to the VCG charging profile, the higher the incentive of the bidders to truth-telling is. By adopting the results from an ordinary combinatorial auction [20], we have the following theorem for the reverse core-selecting AV public transportation auction:

**Theorem 1.** *In an efficient AV public transportation auction, no bidder can increase its utility to more than its VCG utility by coalescing and through shill bidding if and only if it is a core-selecting auction.*

*Proof.* For an efficient auction, we have $\sum_{k\in\mathcal{K}\cup\{0\}} \pi_k \equiv w(\mathcal{K})$ [20], [30]. We require that any coalition $\mathcal{C}$ in $\mathcal{K}$, which can be shills, cannot increase its utility more than its VCG utility. Therefore, we need to have

$$\begin{aligned}\sum_{k\in\mathcal{C}} \pi_k &\leq \sum_{k\in\mathcal{K}\cup\{0\}} \pi_k - \sum_{k\in(\mathcal{K}\setminus\mathcal{C})\cup\{0\}} \pi_k \\ &= w(\mathcal{K}) - w(\mathcal{K}\setminus\mathcal{C}), \forall \mathcal{C} \subseteq \mathcal{K},\end{aligned} \tag{11}$$

which holds if and only if

$$\sum_{k\in(\mathcal{K}\setminus\mathcal{C})\cup\{0\}} \pi_k \geq w(\mathcal{K}\setminus\mathcal{C}), \forall \mathcal{C} \subseteq \mathcal{K}. \tag{12}$$

This condition holds as $\mathcal{K}\setminus\mathcal{C} \subseteq \mathcal{K}$, stating that no blocking coalition exists. Consequently, $\boldsymbol{\pi} \in \mathrm{Core}(\mathcal{K})$. ■

Theorem 1 suggests the theoretical maximum utility that each bidder can achieve through manipulating its bid. However, such unilateral manipulation may change the winners of the auction, resulting in zero utility. Therefore, if the core-selecting charging profile is close enough to the VCG profile, the winning bidders will not perform significant shill bidding. Otherwise, they may actually lose the auction, resulting in zero utilities. Hence the incentive of submitting false bids is suppressed. This inspires the design of the charging rule which will be discussed in Section IV.

Combining Theorem 1 and the definition of core-selecting auction, we have the following corollary:

**Corollary 2.** *A core-selecting mechanism can prevent shill bidding in the proposed AV public transportation auction.*

**Lemma 4.** *For non-splittable and private services, the VCG charging profiles are never in core unless there are multiple bids with the same lowest value.*

*Proof.* For either service type, only one AV can win the auction. When more than one bids have the same value which is the lowest among all, the VCG charging profile is the same of that developed by the first price auction. In such cases, VCG charging profiles are in the core according to Lemma 3.

We then investigate cases where only one bid has the lowest bid value. We use $c^*$ and $k^* \in \mathcal{K}$ to denote the winning service charge and the winning bidder, respectively. The VCG mechanism develops the final charge as follows:

$$\begin{aligned}c^* &= \alpha(\mathcal{K}\setminus k^*) - (\alpha(\mathcal{K}) - x_{k^*}(\mathcal{S}_{k^*}) b_{k^*}(\mathcal{S}_{k^*})) \\ &= \min \sum_{k\in\mathcal{K}\setminus k^*} \sum_{\mathcal{S}\in\mathcal{Q}_k} x_k(\mathcal{S}) b_k(\mathcal{S}).\end{aligned} \tag{13}$$

(13) states that the final service charge is set to the lowest bid value except the auction winner, i.e., the second lowest bid value in the auction. It is always possible to form a coalition $\mathcal{C} \subseteq \mathcal{K}\setminus k^*$ such that $\pi_k = 0, \forall k \in \mathcal{C}$ and $\pi_0 = v_0 - c^*$.

However, if the service charge is adjusted to $c'$ which satisfies $b_{k^*}(\mathcal{S}_{k^*}) \leq c' < c^*$, the new observable surpluses become $\pi'_k = 0, \forall k \in \mathcal{C}$ and $\pi'_0 = v_0 - c' > v_0 - c^* = \pi_0$, where $\boldsymbol{\pi'}$ is the observable surplus profile when $k^*$ charges for $c'$. Therefore, $\mathcal{C}$ is a blocking coalition, and the VCG charge is not in core. ∎

**Theorem 2.** *The only profile in core for non-splittable or private service is the first price auction charge profile.*

*Proof.* Based on Lemma 3, the first price auction profile is in the core. Now we consider any service charge $c^*$ by the sole winner $k^*$ of the auction, such that $c^* > b_{k^*}(\mathcal{S}_{k^*})$. There are more than one bidders participated. According to Lemma 4, there always exists a non-empty coalition blocking $\mathcal{C} \subseteq \mathcal{K} \setminus k^*$ and a new charging price $c' = b_{k^*}(\mathcal{S}_{k^*})$, making $\pi'_k = 0, \forall k \in \mathcal{C}$ and $\pi'_0 = v_0 - c' > v_0 - c^* = \pi_0$.Therefore, no profile with $c^* > b_{k^*}(\mathcal{S}_{k^*})$ is in the core. For $c^* < b_{k^*}(\mathcal{S}_{k^*})$, the auction results do not comply with the rationality assumption of bidders [29], and thus infeasible. Consequently, $c^* = b_{k^*}(\mathcal{S}_{k^*})$ is the only in-core service charge by the auction winner $k^*$ if only one AV can win the auction. ∎

Theorem 2 gives the only in-core service charge profile for non-splittable and private services. Therefore, an efficient charging mechanism should be able to directly find out the only in-core profile, reducing the required computation time for these service types.

## IV. Core-Selecting Charging

Section III gives the theoretical support on the efficacy of core-selecting auctions in preventing shill bidding. In this section, we present the quadratic core-selecting charging rules [29] to select the optimal auction outcomes from the core of an AV public transportation auction. The outcomes preserve the properties of the core while minimizing the final service charge for the customer. According to the definition of $\boldsymbol{\pi}$, given bid values, each observable surplus $\pi_k$ corresponds to a particular service charge $c_k$, $\forall k \in \mathcal{K}$. Therefore, the terms "observable surplus profile" and "charging profile" can be used interchangeably.

### A. Core-Selecting Charging Rule Properties

Although the core-selecting auctions can prevent the formation of any coalition from mutually beneficial re-aggregation [30], they cannot guarantee the absolute truthfulness of bidders. We can approach the truth-telling results through appropriate charging rules. Theorem 1 states that bidders' incentive to truth-telling increases with the decrease in the difference between core-selecting and VCG utilities. Therefore, an efficient core-selecting charging rule minimizes the gap of utilities to motivate truth-telling, and hence improves the seller's utility, or in our case, reduces the service charge.

**Definition 3** (Bidder optimality)**.** $\boldsymbol{\pi}^*$ *is bidder optimal if* $\boldsymbol{\pi}^*$ *is in* $\mathrm{Core}(\mathcal{K})$ *and there does not exist* $\boldsymbol{\pi} \in \mathrm{Core}(\mathcal{K})$ *such that* $\pi_k \leq \pi^*_k$ *for* $k \in \mathcal{K}$ *with strict inequality for at least one* $k$.

**Theorem 3.** *[20] A core-selecting auction provides the minimal incentive for each bidder to misreport its valuation if and only if the auction determines a bidder optimal core profile* $\boldsymbol{\pi}^*$.

*Proof.* The detailed proof is given in [20] and we give an outline here. The result is proved by contradiction. According to Theorem 1, the maximum profit gain induced by a coalition is the Euclidean distance between the core-selecting and VCG charging profiles, where the latter is constant. If the core-selecting auction profile is sub-optimal, there must exist another in-core charging profile that increases the utility of at least one bidder while maintaining others'. This contradicts with the bidder optimality of the original charging profile. ∎

### B. Quadratic Service Charge Minimization

According to Theorem 3, the optimal profiles for bidders in the core represent optimal solutions to the auction, which is mathematically guaranteed optimal. On the one hand, AV operators are entertained as they are requesting just high enough charges to rule out others, and no one can have even larger utilities without undermining others'. On the other hand, the customer is satisfied as no low-charge alternative is apparently available [29]. As such auction outcomes only exist in the core, the problem is translated into finding the optimal solution in the core. In practice, $\mathrm{Core}(\mathcal{K})$ generally contains a large number of bidder optimal solutions. The one with the minimal distance from the VCG charging profile is the most preferable since it can maximize the truth-telling incentive while minimizing the service charge according to Theorems 1 and 3. Similar to [29], we construct a quadratic core-selecting charging rule, due to its corresponding quadratic formulation (will be explained later), to minimize the total service charge and to maximize the truth-telling incentive. The optimal charge can be developed in a two-step process:

*1) First step:* We determine the set of optimal charging profiles from the core with minimum total service charge. Based on Definition 1, we have

$$\sum_{k \in \mathcal{C} \cup \{0\}} \pi_k \geq w(\mathcal{C}), \forall \mathcal{C} \subseteq \mathcal{K}. \tag{14}$$

The charging profile is determined once (3), (4), or (5) is solved. Thus the winning bidders are known. We have

$$\begin{aligned} \sum_{k \in \mathcal{C} \cup \{0\}} \pi_k &= v_0 - \sum_{k \in \mathcal{W}} c_k + \sum_{k \in \mathcal{W} \cap \mathcal{C}} (c_k - b_k(\mathcal{S}_k)) \\ &= v_0 - \sum_{k \in \mathcal{W} \setminus \mathcal{C}} c_k - \sum_{k \in \mathcal{W} \cap \mathcal{C}} b_k(\mathcal{S}_k). \end{aligned} \tag{15}$$

Substituting (15) into (14), we get

$$\sum_{k \in \mathcal{W} \setminus \mathcal{C}} c_k \leq v_0 - \sum_{k \in \mathcal{W} \cap \mathcal{C}} b_k(\mathcal{S}_k) - w(\mathcal{C}), \forall \mathcal{C} \subseteq \mathcal{K}, \tag{16}$$

Then a linear program can be developed to minimize the total service charge:

$$\text{minimize } C^{\mathrm{LP}} = \sum_{k \in \mathcal{K}} c_k \tag{17a}$$

$$\text{subject to } c_k \geq b_k(\mathcal{S}_k), \forall k \in \mathcal{W} \tag{17b}$$

and (16).

This linear program minimizes the total service charge $\sum_{k\in\mathcal{K}} c_k$ from all observable surplus profiles in the core enforced by (16). Since this is a reverse auction, we also incorporate an individual rationality constraint (17b) [29] to ensure that the final charge is no less than the bid value. Therefore, by solving (17), we can obtain the set of optimal charging profiles from the core.

*2) Second step:* To maximize the incentive for truth-telling, we need to find the charging profile, which is closest to the VCG result, with the minimum total service charge obtained in the first step. This can be achieved by minimizing the Euclidean distance of the profiles, resulting in the following quadratic program:

$$\text{minimize} \sum_{k\in\mathcal{K}} (c_k - c_k^{\text{VCG}})^2 \tag{18a}$$

$$\text{subject to} \sum_{k\in\mathcal{K}} c_k = C^{\text{LP}}, \forall k \in \mathcal{K}, \tag{18b}$$

$$\text{(16) and (17b),}$$

where $c_k^{\text{VCG}}$ is the VCG charge of bidder $k$. (18b) guarantees that the result retains the minimal total service charge as obtained in (17).

### C. Implementation

Constraint (16) requires $w(\mathcal{C})$, which is based on a particular $\mathcal{C}$. However, there are total $2^k - 1$ possible $\mathcal{C}$ in the auction. To compute the optimal charging profile, we need to solve the corresponding WDP for each $\mathcal{C}$ in order to construct (17) and (18). It is too computationally expensive in practice. To obtain a near real-time solution, we adopt the core-constraint-generation (CCG) process [19] to reduce the computation burden. The pseudo-code of the process is illustrated in Algorithm 1.

**Algorithm 1** Quadratic Core-Selecting Charging by CCG

1: Solve WDP, find $\mathcal{W}$ and their $\mathcal{S}_k$, determine $c_k^{\text{VCG}}$.
2: Set $t = 0$, initial charge profile $c_k^t = c_k^{\text{VCG}}$.
3: Remove (16) from (17) and (18).
4: **loop**
5:     $t = t + 1$
6:     **for all** $k \in \mathcal{K}, \mathcal{S} \in \mathcal{Q}_k$ **do**
7:         $b_k^t(\mathcal{S}) = c_k^{t-1} + b_k(\mathcal{S}) - b_k(\mathcal{S}_k)$
8:     Solve (3), (4), or (5) with new bids $b_k^t(\mathcal{S})$.
9:     **if** the objective value is not greater than $\sum_{k\in\mathcal{K}} c_k$ **then**
10:         Break loop.
11:     Find the first blocking coalition $\mathcal{C}^t$.
12:     Add constraint $\sum_{k\in\mathcal{W}\setminus\mathcal{C}^t} c_k \geq w(\mathcal{C}^t) + \sum_{k\in\mathcal{C}^t} b_k(\mathcal{S}_k)$ to (17) and (18).
13:     Solve (17) then (18) to get $c_k^t$.
14: $c_k^{t-1}$ is the quadratic core-selecting charge profile.

The algorithm creates a charging profile iteratively. We begin with the VCG profile (Step 1) and remove (16) from (17) and (18) (Step 3). Then in each iteration the algorithm manipulates the submitted bids according to the current charging profile (Step 7), and we compute the auction result (Step 8), which is tested against each possible coalition until a blocking one is found (Step 11). Then we include the corresponding (16) of the blocking coalition back to (17) and (18) (Step 12). These two programs are solved to develop a new charging profile for the next iteration. This process iterates until no blocking coalition exists in the auction, and the developed charging profile is considered as optimal. The efficiency and optimality of CCG is proved in [19], [21].

## V. Performance Evaluation

We perform three simulation tests to assess the efficacy of our proposed core-selecting AV public transportation auctions. Since VCG is the only existing auction mechanism developed for AVPTS in the literature, we largely compare our core-selecting mechanism with VCG. In the first test, we examine the service charges developed by the two mechanisms and this gives us insight into the impact of different parameters on the service charge. The second test investigates the influence of untruthful bids on the core-selecting mechanism. In the third test, we evaluate the computation time required to develop winning charging profiles by the core-selecting mechanism.

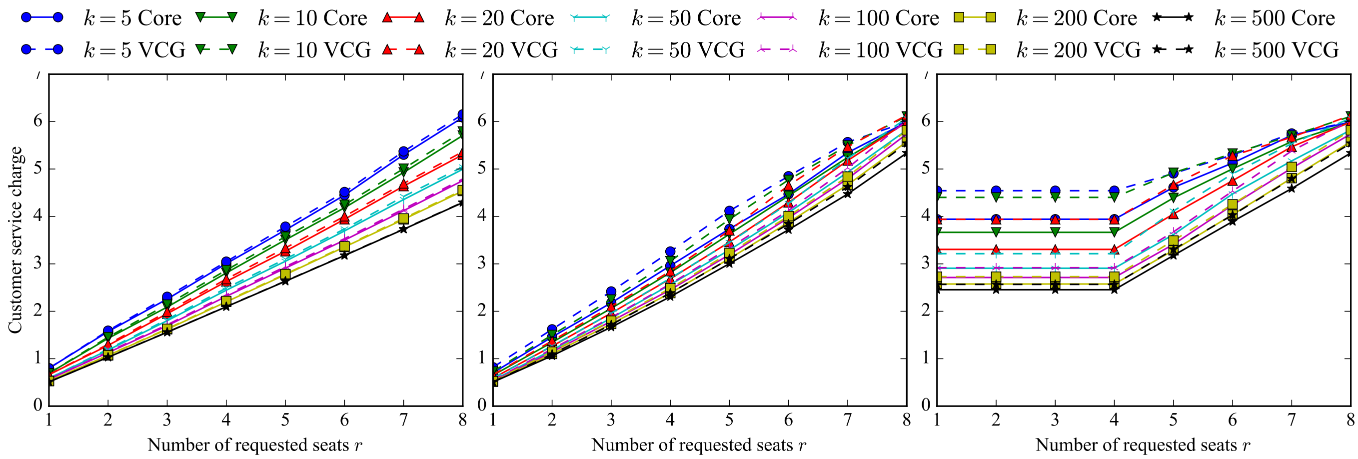
We create random cases with different numbers of bidders for the tests. We generate 100 cases for each $|\mathcal{K}| \in \{5, 10, 20, 50, 100, 200, 500\}$. In each testing case, each AV has seat capacity of a random integer in the range of $[4, 8]$ and the available seats are accordingly generated in the range inclusively from the seat capacity. Both the valuation and the operating cost of each seat occupancy are randomly generated in the range of $[0.5, 1]$. The number of seats required, i.e., $r$, is set in the range of $[1, 8]$.

All tests are performed on a computer with an Intel Core i7-3770 CPU at 3.40 GHz and 12 GB RAM. The testing code is developed with Python 3 under Windows operating system, and the optimization problems are solved with Gurobi [31].

### A. Service Charge

We first investigate the auction results in terms of the final customer charging price. Fig. 2 depicts the VCG and core-selecting charges for the three service types with different numbers of seats requested (i.e., $r$). Each data point is the average produced from the corresponding 100 random cases. In each of the sub-figure, there are 7 sets of lines, each of which contains the results of the core-selecting (solid line) and VCG (dotted line) charges and they correspond to $|\mathcal{K}|$ equal to 5, 10, 20, 50, 100, 200, and 500, respectively. In general, the charges increase with $r$ because more seat occupancies incur higher charge. For the private service, since all seats need to be reserved, the charges remain constant when $r$ is smaller than or equal to the smallest allowed vehicle capacity, i.e., four as defined above.

In general, the service charge for splittable service is lower than those of non-splittable and private services and this accords with [13, Theorem 1]. Moreover, the core-selecting mechanism can produce lower final charges for all service types than VCG auction, and the standard deviations of all data points in Fig. 2 are minuscule. This validates our analytical results given in Theorems 1 and 2, and Lemma 4. In addition,




(a) Splittable service (b) Non-splittable service (c) Private service

Fig. 2. Service charge developed by core-selecting and VCG mechanisms for the three service types.

it can be observed that the core-selecting charges are much more favorable for the private service while its advantage becomes less significant for the splittable service. Lastly, the service charges decrease with the number of bidders. These observations are due to the degree of competition, which is constituted by the number of bids submitted in the auction. Table I shows the average number of bids submitted for all the service types with respect to different $r$ and $\mathcal{K}$. In general, the splittable service has most bids among the three types while the private service has the least. Fewer bids constitute less competition, which makes the bidders easier to manipulate bids for higher profit.

As a result, the core-selecting mechanism can more effectively reduce the service charge, resulting in better performance than the VCG mechanism. Although the charging improvement over VCG for splittable service is not significant, the major merit of the proposed mechanism, as analyzed in Section III, is on its robustness against shill bidding. Meanwhile, despite the fierce competition for splittable service, the core-selecting mechanism can still outperform VCG.

## *B. Truthfulness*

Here we investigate the influence of bidders' truthfulness on the service charge. As mentioned in Section III, no auction mechanisms other than VCG can guarantee truthfulness. However, we can still approach the VCG performance by minimizing the bidder's incentive of reporting bids deviating from their real valuations. Since the splittable service can result in multiple winners, the auction results are more versatile than the other two. Thus we focus on the splittable service here. Consider that a certain number of the bidders do not tell the truth by literally increasing their bids. Specifically, we randomly select 10%, 20%, or 50% of the bidders as untruthful bidders, whose bids are further increased by 10%, 20%, or 30% from their true valuations. Hence there are 9 combinations of untruthful bidders and bid increases, and each combination is evaluated with different $q \in [1, 8]$ in each test case. We compare the relative total winning service charges due to the truthful and untruthful bidders. We define the "increase of charge" as $\left(\sum_{k\in\mathcal{K}} c_k^{\text{new}} - \sum_{k\in\mathcal{K}} c_k\right)/\sum_{k\in\mathcal{K}} c_k$, where $c_k$ and $c_k^{\text{new}}$ refer to the charge of $k$ in the truthful auction and that in the untruthful auction, respectively. The higher the "increase of charge", the larger the amount of overpricing is.

Table II shows the average increase of charge with different requested numbers of seats $r$ in the presence of various levels of truthfulness. We can see that the service charge increases with both the number of untruthfrul bidders and the bid value increase, in which the former plays a more important role. When the percentage of dishonest bidders is relatively small, the increase of charge is not very notable with respect to different levels of bid value increase. There are two reasons for this. First, the core-selecting charging method attempts to minimize the total service charge. The original winners with elevated bids are likely to be replaced by other truthful bidders and thus the untruthful actions would not constitute significant effect on the service charge. Second, as bidders will not know if they are among the winners of the auction, their service charge increase may not always undermine the system performance. This supports our previous theoretical analyses in Section IV. When the fraction of dishonest bidders increases to 50%, the auction becomes sensitive to bid value increase. In such cases, it is very likely that the winners have already increased their bid values, and their amount of increase impacts significantly on the increase of charge. In the worst case with 50% of the bidders reporting 30% higher service charges, the core-selecting auction results in 4.5%–8% of increase in charge, despite that on average the bids experience a 15% increase. This shows that the core-selecting mechanism is robust against untruthful bidders.

Next, we compare the performance of core-selecting and VCG mechanisms. In Table II, those results with smaller increase of charge is highlighted in bold. It can be observed that the performance of the two mechanisms is similar. In most cases, the core-selecting mechanism slightly outperforms VCG. This is due to the fact that the core-selecting mecha-

TABLE I
AVERAGE NUMBER OF BIDS

| $\lvert\mathcal{K}\rvert$ | Service | Number of requested seats $r$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 5 | Splittable | 17.18 | 17.18 | 17.18 | 17.18 | 17.18 | 17.18 | 17.18 | 17.18 |
| | Non-splittable | 17.18 | 12.18 | 8.19 | 5.02 | 2.71 | 1.34 | 0.52 | 0.15 |
| | Private | 0.95 | 0.95 | 0.95 | 0.95 | 0.67 | 0.47 | 0.26 | 0.15 |
| 10 | Splittable | 35.32 | 35.32 | 35.32 | 35.32 | 35.32 | 35.32 | 35.32 | 35.32 |
| | Non-splittable | 35.32 | 25.32 | 17.12 | 10.5 | 5.73 | 2.87 | 1.18 | 0.31 |
| | Private | 1.98 | 1.98 | 1.98 | 1.98 | 1.4 | 0.99 | 0.6 | 0.31 |
| 20 | Splittable | 70.25 | 70.25 | 70.25 | 70.25 | 70.25 | 70.25 | 70.25 | 70.25 |
| | Non-splittable | 70.25 | 50.25 | 33.84 | 20.77 | 11.35 | 5.58 | 2.24 | 0.55 |
| | Private | 3.87 | 3.87 | 3.87 | 3.87 | 2.79 | 1.94 | 1.17 | 0.55 |
| 50 | Splittable | 175.16 | 175.16 | 175.16 | 175.16 | 175.16 | 175.16 | 175.16 | 175.16 |
| | Non-splittable | 175.16 | 125.16 | 84.11 | 51.51 | 28.07 | 13.56 | 5.54 | 1.35 |
| | Private | 9.19 | 9.19 | 9.19 | 9.19 | 6.59 | 4.43 | 2.97 | 1.35 |
| 100 | Splittable | 352.19 | 352.19 | 352.19 | 352.19 | 352.19 | 352.19 | 352.19 | 352.19 |
| | Non-splittable | 352.19 | 252.19 | 169.98 | 104.49 | 56.76 | 27.52 | 10.92 | 2.62 |
| | Private | 18.34 | 18.34 | 18.34 | 18.34 | 13.16 | 8.92 | 5.7 | 2.62 |
| 200 | Splittable | 702.67 | 702.67 | 702.67 | 702.67 | 702.67 | 702.67 | 702.67 | 702.67 |
| | Non-splittable | 702.67 | 502.67 | 338.32 | 208.44 | 113.79 | 55.1 | 21.69 | 5.25 |
| | Private | 36.41 | 36.41 | 36.41 | 36.41 | 26.18 | 18.18 | 11.52 | 5.25 |
| 500 | Splittable | 1753.36 | 1753.36 | 1753.36 | 1753.36 | 1753.36 | 1753.36 | 1753.36 | 1753.36 |
| | Non-splittable | 1753.36 | 1253.36 | 841.85 | 517.55 | 282.41 | 135.44 | 52.25 | 12.34 |
| | Private | 87.51 | 87.51 | 87.51 | 87.51 | 63.5 | 43.6 | 27.41 | 12.34 |

TABLE II
INCREASE OF CHARGE WITH LEVELS OF TRUTHFULNESS

| Untruthful | Increase | Mechanism | Request size $r$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 10% | 10% | Core-selecting | **0.61%** | 0.52% | **0.44%** | **0.58%** | **0.79%** | 0.65% | **0.63%** | **0.66%** |
| | | VCG | 0.61% | **0.47%** | 0.46% | 0.67% | 0.88% | **0.57%** | 0.65% | 0.69% |
| | 20% | Core-selecting | **0.65%** | 0.72% | **0.49%** | **0.34%** | **0.72%** | **0.89%** | 0.55% | **0.64%** |
| | | VCG | 0.65% | **0.53%** | 0.73% | 0.39% | 0.75% | 0.99% | **0.42%** | 0.77% |
| | 30% | Core-selecting | **0.77%** | **0.73%** | **0.45%** | **0.75%** | **0.71%** | **0.88%** | **0.79%** | **0.83%** |
| | | VCG | **0.77%** | 0.77% | 0.49% | 0.87% | 0.87% | 0.89% | 0.94% | 1.10% |
| 20% | 10% | Core-selecting | 1.63% | **1.14%** | **1.42%** | **0.85%** | **1.32%** | **1.22%** | 1.24% | **0.98%** |
| | | VCG | **1.63%** | 1.33% | 1.53% | 0.90% | 1.49% | 1.23% | **1.16%** | 1.03% |
| | 20% | Core-selecting | 1.49% | **1.52%** | **1.75%** | 1.07% | **1.32%** | **1.55%** | 1.33% | **1.35%** |
| | | VCG | **1.49%** | 1.68% | 2.06% | **1.03%** | 1.38% | 1.61% | **1.22%** | 1.45% |
| | 30% | Core-selecting | 2.69% | **1.55%** | **1.60%** | **1.36%** | **1.11%** | 1.99% | **1.50%** | **1.15%** |
| | | VCG | **2.69%** | 1.67% | 2.12% | 1.45% | 1.12% | **1.96%** | 1.59% | 1.22% |
| 50% | 10% | Core-selecting | **4.14%** | 3.30% | **3.98%** | **3.56%** | 3.60% | 3.56% | **3.22%** | **3.24%** |
| | | VCG | 4.14% | **3.20%** | 4.24% | 3.70% | **3.36%** | **3.54%** | 3.23% | 3.33% |
| | 20% | Core-selecting | **5.84%** | **4.96%** | **4.75%** | **4.21%** | 4.65% | **4.16%** | 4.38% | **4.44%** |
| | | VCG | **5.84%** | 5.34% | 5.11% | 4.34% | **4.62%** | 4.32% | **4.27%** | 4.61% |
| | 30% | Core-selecting | **8.09%** | **5.55%** | **5.31%** | **4.97%** | **4.59%** | **5.26%** | **4.48%** | **4.52%** |
| | | VCG | 8.09% | 5.64% | 5.74% | 5.20% | 5.00% | 5.61% | 4.57% | 4.71% |

nism can maximize customers' utility as analyzed in Section III. Although the core-selecting mechanism cannot guarantee bidder truthfulness like VCG, it can still efficiently reduce the influence of dishonest bidders on the final service charge. The benefit of minimized service charge, as illustrated in Section V-A, can in general outweigh the hindrance of suppressing untruthfulness. Therefore, core-selecting mechanism can generally develop satisfactory service charge profiles even in the presence of overpriced bids, and can sometimes outperform the truth-guaranteeing VCG mechanism.

## C. Computation Time

The computation time for determining the winners and their respective service charges is also of importance to the pragmatic implementation of our proposed auction in practice. A shorter computation time will reduce customers' waiting time, resulting in better quality of service. Fig. 3 shows the

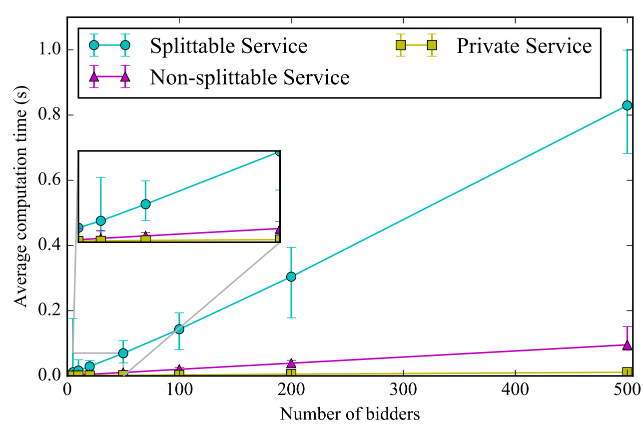



Fig. 3. Computation time of three services with various number of bidders.

average and extrema of the computation time for the three service types. We can see that all service types experience linear computation time increase with the number of bidders. Even in the worst case, it only takes less than one second to compute the result. This indicates that our mechanism is scalable and practical. In addition, the splittable and non-splittable services require much less than the splittable service. This can be credited to the significantly smaller solution spaces of their WDPs and less computationally demanding core-selecting charging process.

## VI. Conclusion

In a multi-tenant AVPTS, the pricing process is used to settle the service charge when multiple operators compete to accommodate a transport request. The VCG auction mechanism, adopted in the original design [12], suffers from some significant drawbacks such as vulnerable to shill-bidding and coalitions, rendering the VCG design impractical. In this work, by looking into the core of the auctions, we construct the core-selecting AV public transportation auctions to overcome the problems brought by VCG. We first formulate the reverse combinatorial AV public transportation auctions to model the pricing processes of the three transportation service types. Then we investigate the core of these auctions, which demonstrates the non-blocking properties of auction results. Utilizing this feature, we construct a core-selecting charging rule to determine the service charges of the winning vehicles. We verify our analytical results with extensive simulations. The proposed core-selecting mechanism can result in lower total service charges than VCG mechanism and it can also almost preserve VCG's unique ability against untruthful bidders. Last but not least, the short computational speed of the core-selecting mechanism can result in a practical pricing solution for AVPTS for real-world implementation.

The future research is two-fold. First, it is of interest to investigate other auction mechanisms for AVPTS pricing process. While the proposed core-selecting mechanism can successfully prevent shill-bids, other mechanisms may possibly provides other advantages. Second, this work follows our previous work [13] in which requests are handled sequentially. Second, it remains unknown how multiple requests will influence the charging price and service time of the pricing process. In addition, we will investigate suitable auctioning mechanisms for such multi-tenant multi-request process.

## Appendix A
## Vickrey-Clarke-Groves and Shill-bids

In this appendix, we briefly discuss the pricing mechanism of VCG in a reversed combinatorial auction. This mechanism was employed in the pricing process of AVPTS in [13].

VCG [14]–[16] suggests that each winning bidder in an auction should charge the amount of "damage" introduced to all bidders. Let $\mathcal{Q}$ be the set of auctioned items and $\mathcal{K}$ be the set of bidders/operators who sell the items. Let $b_k(\mathcal{S})$ be the reported operational cost (social value) for $k \in \mathcal{K}$ to provide $\mathcal{S} \subseteq \mathcal{Q}$, and $x_k(\mathcal{S})$ be a binary variable indicating whether $k$ wins with $\mathcal{S}$ ($x_k(\mathcal{S}) = 1$) or not ($x_k(\mathcal{S}) = 0$). Accordingly, the total "social value" after the auction can be calculated as $\sum_{k\in\mathcal{K}} \sum_{\mathcal{S}\subseteq\mathcal{Q}} x_k(\mathcal{S}) b_k(\mathcal{S})$.

For a winning $k^* \in \mathcal{K}$, instead of charging $b_{k^*}(\mathcal{S}^*)$ for $\mathcal{S}^*$, the service charge is determined by the "social cost" of their winning incurred by other operators:

$$\sum_{k\in\mathcal{K}} \sum_{\mathcal{S}\subseteq\mathcal{Q}} x'_k(\mathcal{S}) b_k(\mathcal{S}) - [\sum_{k\in\mathcal{K}} \sum_{\mathcal{S}\subseteq\mathcal{Q}} x_k(\mathcal{S}) b_k(\mathcal{S}) - b_{k^*}(\mathcal{S}^*)] \tag{19}$$

where $x'_k(\mathcal{S})$ is the winning result when $k$ does not place a bid for $\mathcal{S}$. The first term in (19) denotes the total social value in the new auction and the second denotes the social value in the original auction except $b_{k^*}(\mathcal{S}^*)$.

We consider an example for illustration: A customer submits a transportation request consists of two passengers A and B[3]. A sole operator X places a bid to serve both passengers at a charge of \$10. So he will win the auction and serve both passengers at \$10.

Meanwhile, X may impersonate two other fake operators or form a coalition with two other real operators, denoted as Y and Z. If so, X still bids to serve A and B for \$10, Y bids to serve A for \$10, and Z bids to serve B for \$10. In this case, X still wins the auction. Meanwhile, the service charge of X is set according to (19). In this auction, the total social value for Y and Z is \$0, since neither of them wins. If X is removed from the auction, the winners will be Y and Z, and the total social value will become \$20. Therefore, X can charge \$20 (total value in the new auction)−\$0 (total value except X in the original auction=\$20. This example shows the mechanism of VCG pricing, and its vulnerability against shill-bids.

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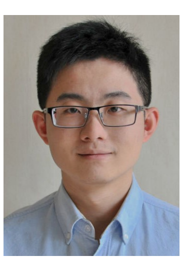

**James J.Q. Yu** (S'11–M'15) received the B.Eng. and Ph.D. degree in Electrical and Electronic Engineering from the University of Hong Kong, Pokfulam, Hong Kong, in 2011 and 2015, respectively. He is currently an honorary assistant professor and post-doctoral fellow at the Department of Electrical and Electronic Engineering, The University of Hong Kong. He is also the Chief Research Consultant of GWGrid Inc. and Fano Labs. His research interests include smart city technologies, deep learning and big data industrial applications, power stability analysis, and evolutionary computation.

**Albert Y.S. Lam** (S'03–M'10–SM'16) received the BEng degree (First Class Honors) in Information Engineering and the PhD degree in Electrical and Electronic Engineering from the University of Hong Kong (HKU), Hong Kong, in 2005 and 2010, respectively. He was a postdoctoral scholar at the Department of Electrical Engineering and Computer Sciences of University of California, Berkeley, CA, USA, in 2010–12. Now he is the Chief Scientist and the acting Chief Technology Officer at Fano Labs, and a honorary assistant professor at the Department of Electrical and Electronic Engineering of HKU. He is a Croucher research fellow. His research interests include optimization theory and algorithms, evolutionary computation, smart grid, and smart city.